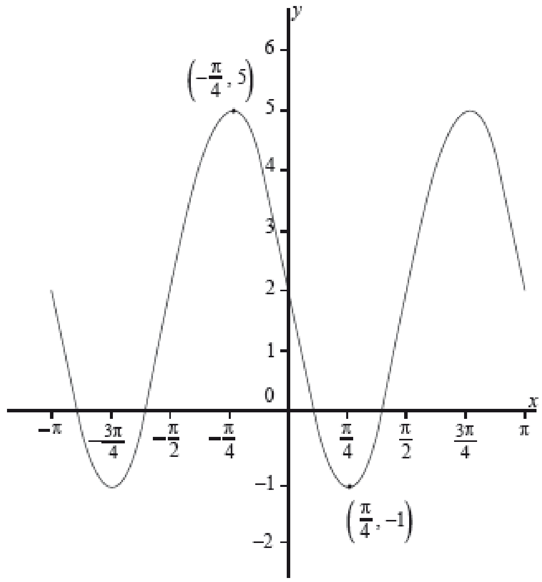
HL Paper 2

A function is defined by $f(x) = A\sin(Bx) + C, \ -\pi \le x \le \pi$, where $A,\ B,\ C \in \mathbb{Z}$. The following diagram represents the graph of $y = f(x)$.



a. Find the value of [4]

(i) A;

(ii) B;

(iii) C.

b. Solve $f(x) = 3$ for $0 \le x \le \pi$. [2]

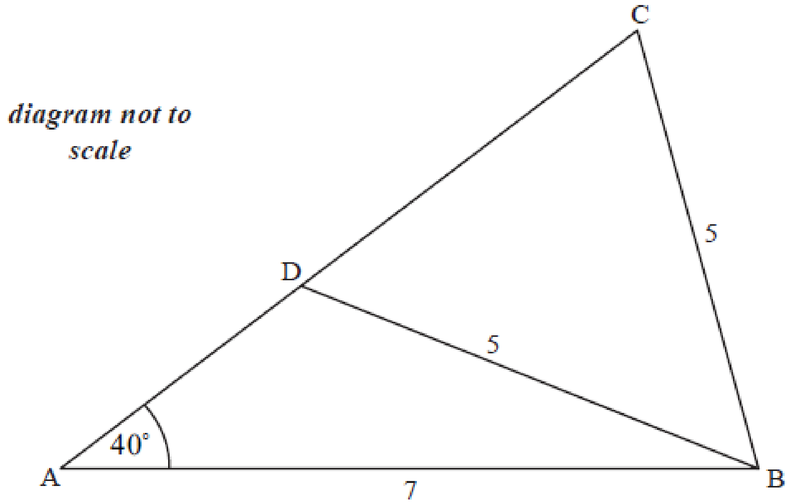
Consider the function f defined by $f(x) = 3x\arccos(x)$ where $-1 \leqslant x \leqslant 1$.

a. Sketch the graph of f indicating clearly any intercepts with the axes and the coordinates of any local maximum or minimum points. [3]

b. State the range of f. [2]

c. Solve the inequality $|3x\arccos(x)| > 1$. [4]

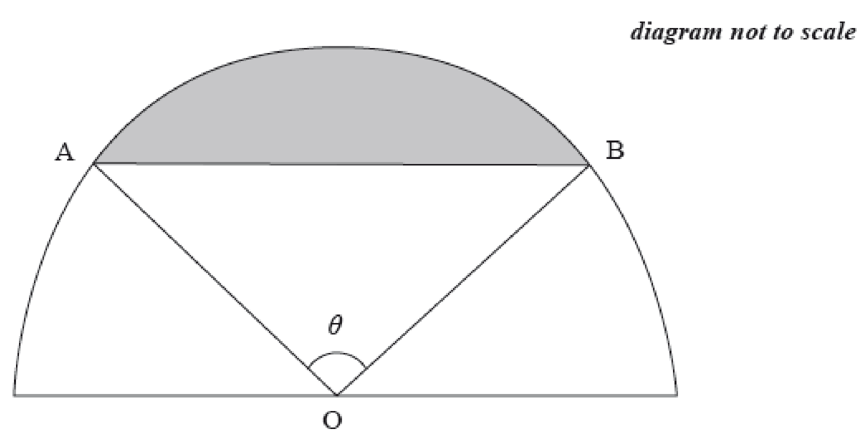
Given ΔABC, with lengths shown in the diagram below, find the length of the line segment [CD].



Consider triangle ABC with $B\hat{A}C = 37.8°$, AB = 8.75 and BC = 6 .

Find AC.

The diagram below shows a semi-circle of diameter 20 cm, centre O and two points A and B such that $A\hat{O}B = \theta$, where θ is in radians.



a. Show that the shaded area can be expressed as $50\theta - 50\sin\theta$. [2]

b. Find the value of θ for which the shaded area is equal to half that of the unshaded area, giving your answer correct to four significant figures. [3]

In triangle PQR, $\mathrm{PR} = 12$ cm, $\mathrm{QR} = p$ cm, $\mathrm{PQ} = r$ cm and $\mathrm{Q\hat{P}R} = 30°$.

Consider the possible triangles with $\mathrm{QR} = 8$ cm.

Consider the case where p, the length of QR is not fixed at 8 cm.

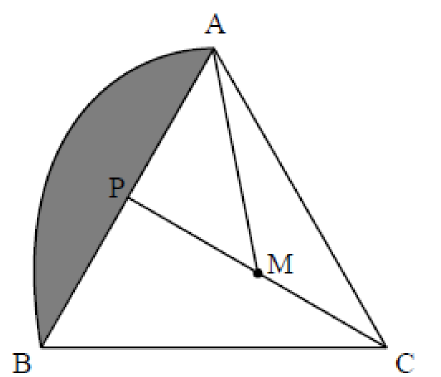
a. Use the cosine rule to show that $r^2 - 12\sqrt{3}r + 144 - p^2 = 0$. [2]

b. Calculate the two corresponding values of PQ. [3]

c. Hence, find the area of the smaller triangle. [3]

d. Determine the range of values of p for which it is possible to form two triangles. [7]

Consider the following diagram.



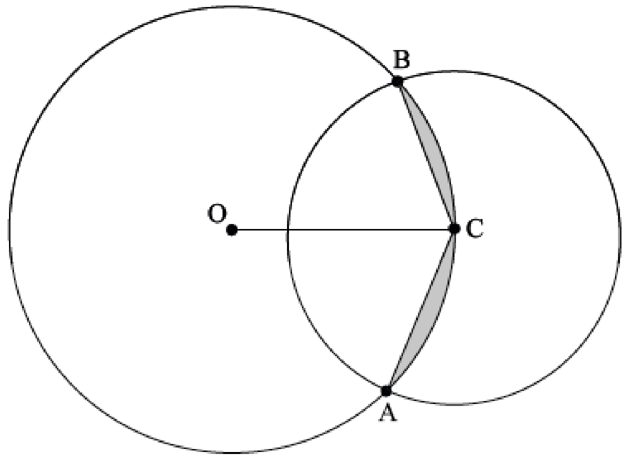
The sides of the equilateral triangle ABC have lengths 1 m. The midpoint of [AB] is denoted by P. The circular arc AB has centre, M, the midpoint of [CP].

a.i. Find AM. [3]

a.ii. Find $\mathrm{A\hat{M}P}$ in radians. [2]

b. Find the area of the shaded region. [3]

The following diagram shows two intersecting circles of radii 4 cm and 3 cm. The centre C of the smaller circle lies on the circumference of the bigger circle. O is the centre of the bigger circle and the two circles intersect at points A and B.



Find:

(a) $\mathrm{B\hat{O}C}$;

(b) the area of the shaded region.

Triangle ABC has AB = 5 cm, BC = 6 cm and area 10 cm^2.

(a) Find $\sin \hat{B}$.

(b) **Hence**, find the two possible values of AC, giving your answers correct to two decimal places.

In a triangle ABC, $\hat{A} = 35^\circ$, BC = 4 cm and AC = 6.5 cm. Find the possible values of $\hat{B}$ and the corresponding values of AB.

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a. Given that $\arctan \frac{1}{2} - \arctan \frac{1}{3} = \arctan a,\ a \in \mathbb{Q}^+$, find the value of a. [3]

b. Hence, or otherwise, solve the equation $\arcsin x = \arctan a$. [2]

Consider the triangle PQR where $\mathrm{Q\hat{P}R} = 30^\circ$, $\mathrm{PQ} = (x+2)$ cm and $\mathrm{PR} = (5-x)^2$ cm, where $-2 < x < 5$.

a. Show that the area, A cm^2, of the triangle is given by $A = \frac{1}{4}(x^3 - 8x^2 + 5x + 50)$. [2]

b. (i) State $\frac{\mathrm{d}A}{\mathrm{d}x}$. [3]

(ii) Verify that $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ when $x = \frac{1}{3}$.

c. (i) Find $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}$ and hence justify that $x = \frac{1}{3}$ gives the maximum area of triangle PQR. [7]

(ii) State the maximum area of triangle PQR.

(iii) Find QR when the area of triangle PQR is a maximum.

The function $f(x) = 3\sin x + 4\cos x$ is defined for $0 < x < 2\pi$.

a. Write down the coordinates of the minimum point on the graph of f . [1]

b. The points $\text{P}(p,\ 3)$ and $\text{Q}(q,\ 3)$, $q > p$, lie on the graph of $y = f(x)$. [2]

Find p and q .

c. Find the coordinates of the point, on $y = f(x)$, where the gradient of the graph is 3. [4]

d. Find the coordinates of the point of intersection of the normals to the graph at the points P and Q. [7]

A circle of radius 4 cm , centre O , is cut by a chord [AB] of length 6 cm.

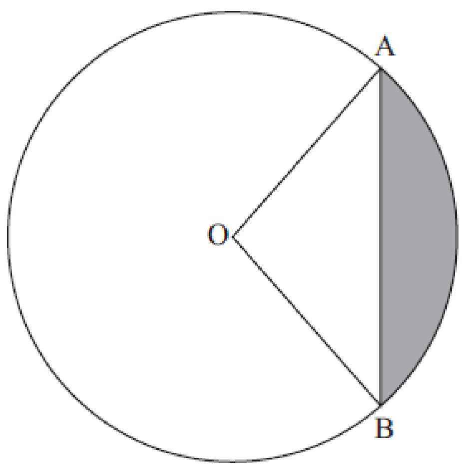


diagram not to scale

a. Find $\text{A}\hat{\text{O}}\text{B}$, expressing your answer in radians correct to four significant figures. [2]

b. Determine the area of the shaded region. [3]

Consider the triangle ABC where $\text{B}\hat{\text{A}}\text{C} = 70^\circ$, AB = 8 cm and AC = 7 cm. The point D on the side BC is such that $\frac{\text{BD}}{\text{DC}} = 2$.

Determine the length of AD.

The depth, $h(t)$ metres, of water at the entrance to a harbour at t hours after midnight on a particular day is given by

$$h(t) = 8 + 4\sin\left(\frac{\pi t}{6}\right),\ 0 \leqslant t \leqslant 24.$$

(a) Find the maximum depth and the minimum depth of the water.

(b) Find the values of t for which $h(t) \geqslant 8$.

Farmer Bill owns a rectangular field, 10 m by 4 m. Bill attaches a rope to a wooden post at one corner of his field, and attaches the other end to his goat Gruff.

a. Given that the rope is 5 m long, calculate the percentage of Bill's field that Gruff is able to graze. Give your answer correct to the nearest integer. [4]

b. Bill replaces Gruff's rope with another, this time of length $a,\ 4 < a < 10$, so that Gruff can now graze exactly one half of Bill's field. [4]

Show that a satisfies the equation

$$a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40.$$

c. Find the value of a. [2]

In triangle ABC, $\mathrm{AB} = 5\ \mathrm{cm}$, $\mathrm{BC} = 12\ \mathrm{cm}$ and $\mathrm{A\hat{B}C} = 100^\circ$.

a. Find the area of the triangle. [2]

b. Find AC. [2]

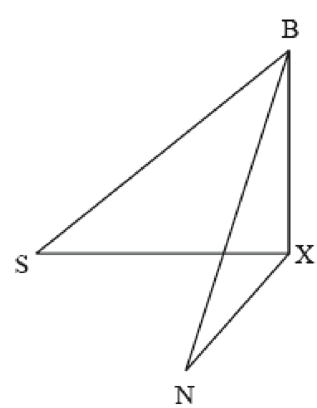
Barry is at the top of a cliff, standing 80 m above sea level, and observes two yachts in the sea.

"*Seaview*" (S) is at an angle of depression of 25°.

"*Nauti Buoy*" (N) is at an angle of depression of 35°.

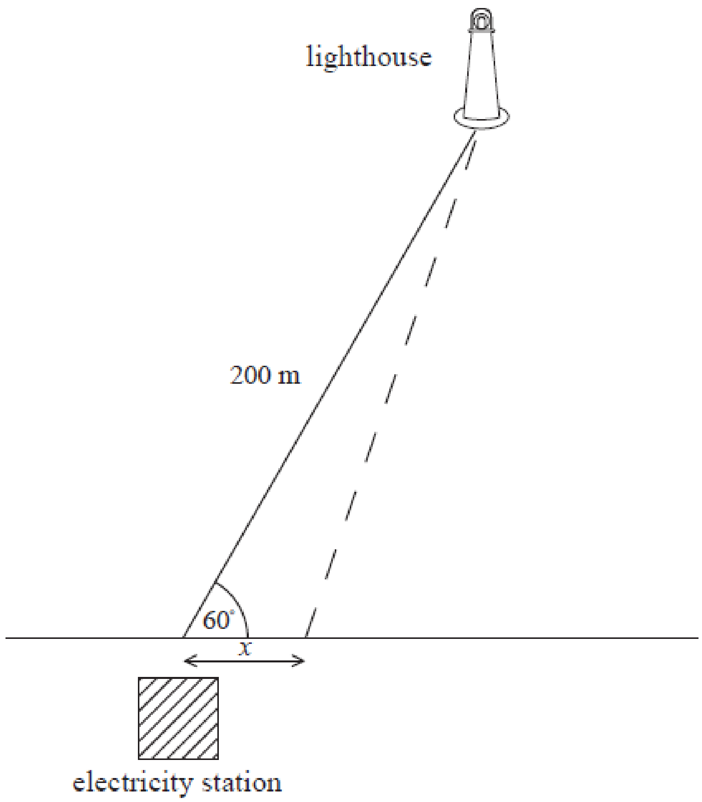
The following three dimensional diagram shows Barry and the two yachts at S and N.

X lies at the foot of the cliff and angle $\mathrm{SXN} = 70°$.



Find, to 3 significant figures, the distance between the two yachts.

An electricity station is on the edge of a straight coastline. A lighthouse is located in the sea 200 m from the electricity station. The angle between the coastline and the line joining the lighthouse with the electricity station is 60°. A cable needs to be laid connecting the lighthouse to the electricity station. It is decided to lay the cable in a straight line to the coast and then along the coast to the electricity station. The length of cable laid along the coastline is x metres. This information is illustrated in the diagram below.



The cost of laying the cable along the sea bed is US$80 per metre, and the cost of laying it on land is US$20 per metre.

a. Find, in terms of x, an expression for the cost of laying the cable. [4]

b. Find the value of x, to the nearest metre, such that this cost is minimized. [2]

The vertices of an equilateral triangle, with perimeter P and area A , lie on a circle with radius r . Find an expression for $\frac{P}{A}$ in the form $\frac{k}{r}$, where $k \in \mathbb{Z}^{+}$.

Consider the function $f(x) = 2\sin^2 x + 7\sin 2x + \tan x - 9,\ 0 \leqslant x < \frac{\pi}{2}$.

Let $u = \tan x$.

a.i. Determine an expression for $f'(x)$ in terms of x. [2]

a.ii. Sketch a graph of $y = f'(x)$ for $0 \leqslant x < \frac{\pi}{2}$. [4]

a.iii. Find the x-coordinate(s) of the point(s) of inflexion of the graph of $y = f(x)$, labelling these clearly on the graph of $y = f'(x)$. [2]

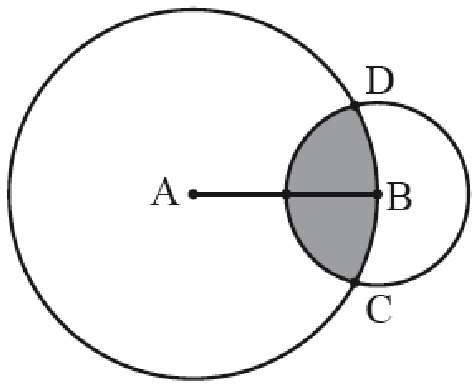
b.i. Express $\sin x$ in terms of μ. [2]

b.ii. Express $\sin 2x$ in terms of u. [3]

b.iii. Hence show that $f(x) = 0$ can be expressed as $u^3 - 7u^2 + 15u - 9 = 0$. [2]

c. Solve the equation $f(x) = 0$, giving your answers in the form $\arctan k$ where $k \in \mathbb{Z}$. [3]

The diagram shows two circles with centres at the points A and B and radii $2r$ and r, respectively. The point B lies on the circle with centre A. The circles intersect at the points C and D.



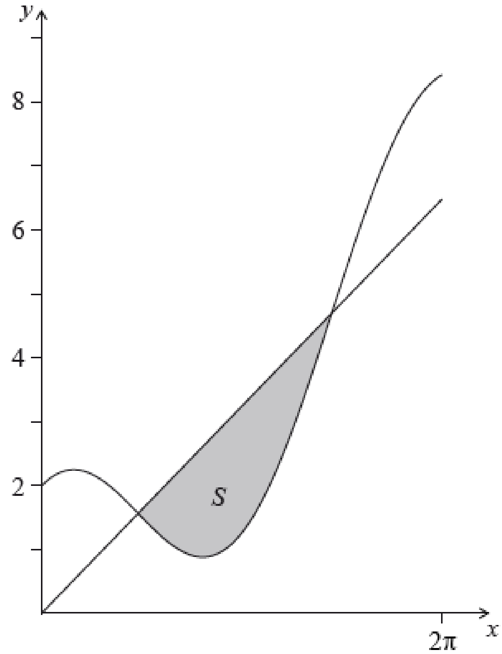
Let α be the measure of the angle CAD and θ be the measure of the angle CBD in radians.

a. Find an expression for the shaded area in terms of α, θ and r. [3]

b. Show that $\alpha = 4\arcsin\frac{1}{4}$. [2]

c. Hence find the value of r given that the shaded area is equal to 4. [3]

The shaded region S is enclosed between the curve $y = x + 2\cos x$, for $0 \leqslant x \leqslant 2\pi$, and the line $y = x$, as shown in the diagram below.



a. Find the coordinates of the points where the line meets the curve. [3]

b. The region S is rotated by 2π about the x-axis to generate a solid. [5]

(i) Write down an integral that represents the volume V of the solid.

(ii) Find the volume V.

In a triangle ABC, $\mathrm{AB} = 4\text{ cm}$, $\mathrm{BC} = 3\text{ cm}$ and $\mathrm{B\hat{A}C} = \frac{\pi}{9}$.

a. Use the cosine rule to find the two possible values for AC. [5]

b. Find the difference between the areas of the two possible triangles ABC. [3]

Let $z = r(\cos\alpha + \mathrm{i}\sin\alpha)$, where α is measured in degrees, be the solution of $z^5 - 1 = 0$ which has the smallest positive argument.

a. (i) Use the binomial theorem to expand $(\cos\theta + \mathrm{i}\sin\theta)^5$. [6]

(ii) Hence use De Moivre's theorem to prove

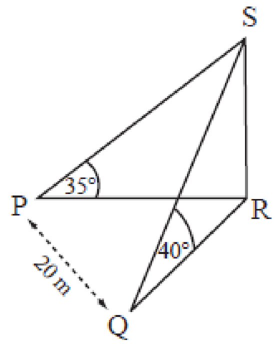
$$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta.$$

(iii) State a similar expression for $\cos 5\theta$ in terms of $\cos\theta$ and $\sin\theta$.

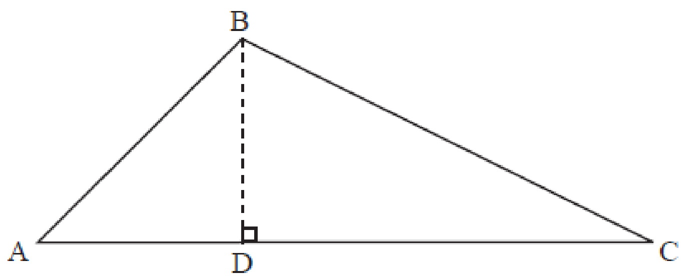
b. Find the value of r and the value of α. [4]

c. Using (a) (ii) and your answer from (b) show that $16\sin^4\alpha - 20\sin^2\alpha + 5 = 0$. [4]

d. Hence express $\sin 72^\circ$ in the form $\frac{\sqrt{a+b\sqrt{c}}}{d}$ where $a,\ b,\ c,\ d \in \mathbb{Z}$. [5]



Part A In triangle ABC, BC = a , AC = b , AB = c and [BD] is perpendicular to [AC]. [12]



(a) Show that $\mathrm{CD} = b - c\cos A$.

(b) **Hence**, by using Pythagoras' Theorem in the triangle BCD, prove the cosine rule for the triangle ABC.

(c) If $\mathrm{A\hat{B}C} = 60^\circ$, use the cosine rule to show that $c = \frac{1}{2}a \pm \sqrt{b^2 - \frac{3}{4}a^2}$.

Part B The above three dimensional diagram shows the points P and Q which are respectively west and south-west of the base R of a vertical flagpole RS on horizontal ground. The angles of elevation of the top S of the flagpole from P and Q are respectively 25° and 40° , and PQ = 20 m . [8]

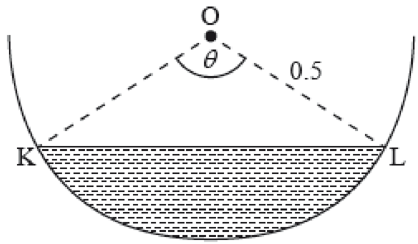
Determine the height of the flagpole.

Let $f(x) = \tan(x + \pi)\cos\left(x - \frac{\pi}{2}\right)$ where $0 < x < \frac{\pi}{2}$.

Express $f(x)$ in terms of sin$x$ and cos$x$.

ABCD is a quadrilateral where $\mathrm{AB} = 6.5,\ \mathrm{BC} = 9.1,\ \mathrm{CD} = 10.4,\ \mathrm{DA} = 7.8$ and $\mathrm{C\hat{D}A} = 90^\circ$. Find $\mathrm{A\hat{B}C}$, giving your answer correct to the nearest degree.

A water trough which is 10 metres long has a uniform cross-section in the shape of a semicircle with radius 0.5 metres. It is partly filled with water as shown in the following diagram of the cross-section. The centre of the circle is O and the angle KOL is θ radians.



The volume of water is increasing at a constant rate of $0.0008\ \mathrm{m^3 s^{-1}}$.

a. Find an expression for the volume of water $V\ (\mathrm{m}^3)$ in the trough in terms of θ. [3]

b. Calculate $\frac{\mathrm{d}\theta}{\mathrm{d}t}$ when $\theta = \frac{\pi}{3}$. [4]

Triangle ABC has area $21\ \mathrm{cm}^2$. The sides AB and AC have lengths 6 cm and 11 cm respectively. Find the two possible lengths of the side BC.

Consider the planes $\pi_1 : x - 2y - 3z = 2$ and $\pi_2 : 2x - y - z = k$.

a. Find the angle between the planes $\pi_1$and π_2 . [4]

b. The planes π_1 and π_2 intersect in the line L_1 . Show that the vector equation of [5]

L_1 is $r = \begin{pmatrix} 0 \\ 2-3k \\ 2k-2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$

c. The line L_2 has Cartesian equation $5 - x = y + 3 = 2 - 2z$. The lines L_1 and L_2 intersect at a point X. Find the coordinates of X. [5]

d. Determine a Cartesian equation of the plane π_3 containing both lines L_1 and L_2 . [5]

e. Let Y be a point on L_1 and Z be a point on L_2 such that XY is perpendicular to YZ and the area of the triangle XYZ is 3. Find the perimeter of the triangle XYZ. [5]

In triangle ABC,

$3\sin B + 4\cos C = 6$ and

$4\sin C + 3\cos B = 1$.

a. Show that $\sin(B + C) = \frac{1}{2}$. [6]

b. Robert conjectures that $\mathrm{C\hat{A}B}$ can have two possible values. [5]

Show that Robert's conjecture is incorrect by proving that $\mathrm{C\hat{A}B}$ has only one possible value.

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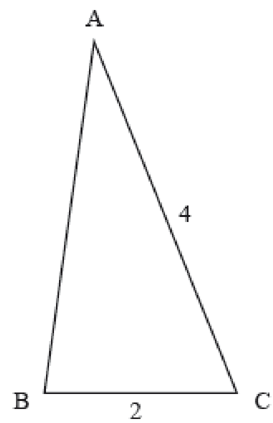
a. Find the set of values of k that satisfy the inequality $k^2 - k - 12 < 0$. [2]

b. The triangle ABC is shown in the following diagram. Given that $\cos B < \frac{1}{4}$, find the range of possible values for AB. [4]



A triangle ABC has $\hat{A} = 50°$, $\mathrm{AB} = 7\text{ cm}$ and $\mathrm{BC} = 6\text{ cm}$. Find the area of the triangle given that it is smaller than 10 cm^2.

A rectangle is drawn around a sector of a circle as shown. If the angle of the sector is 1 radian and the area of the sector is 7 cm^2, find the dimensions of the rectangle, giving your answers to the nearest millimetre.

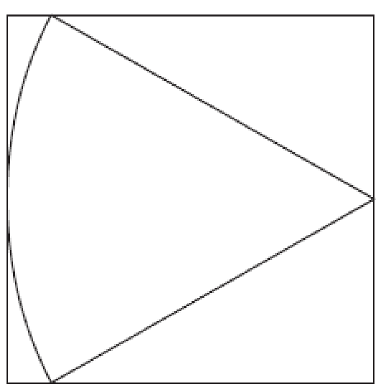
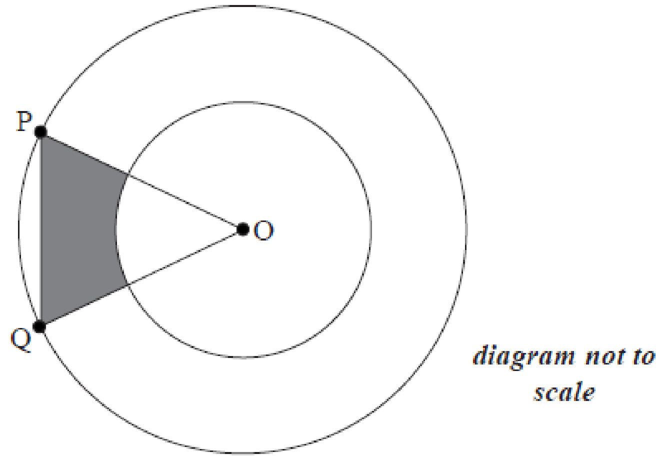


diagram not to scale

The diagram below shows two concentric circles with centre O and radii 2 cm and 4 cm.

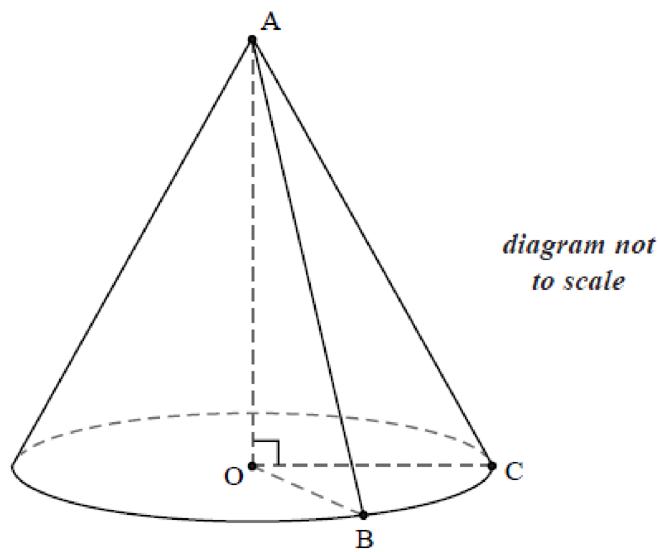
The points P and Q lie on the larger circle and $\mathrm{P\hat{O}Q} = x$, where $0 < x < \frac{\pi}{2}$.



(a) Show that the area of the shaded region is $8\sin x - 2x$.

(b) Find the maximum area of the shaded region.

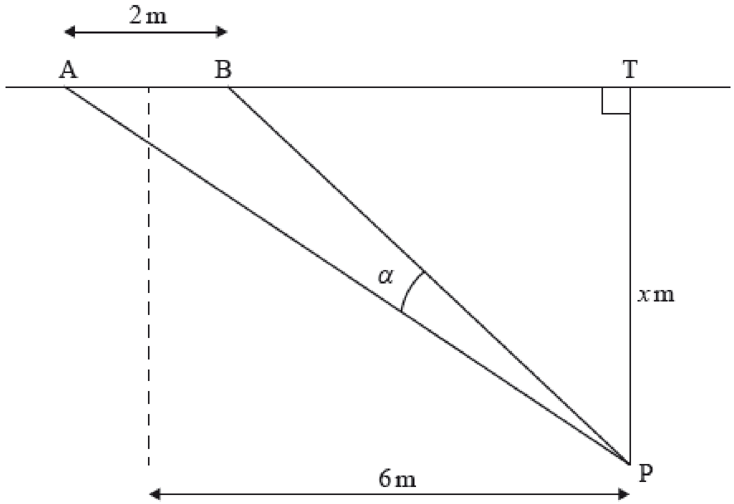
In the right circular cone below, O is the centre of the base which has radius 6 cm. The points B and C are on the circumference of the base of the cone. The height AO of the cone is 8 cm and the angle $\mathrm{B\hat{O}C}$ is 60°.



Calculate the size of the angle $\mathrm{B\hat{A}C}$.

Points A , B and T lie on a line on an indoor soccer field. The goal, [AB] , is 2 metres wide. A player situated at point P kicks a ball at the goal. [PT] is perpendicular to (AB) and is 6 metres from a parallel line through the centre of [AB] . Let PT be x metros and let $\alpha = \mathrm{A\hat{P}B}$ measured in degrees.

Assume that the ball travels along the floor.



The maximum for $\tan\alpha$ gives the maximum for α.

a. Find the value of α when $x = 10$. [4]

b. Show that $\tan\alpha = \frac{2x}{x^2+35}$. [4]

c. (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}(\tan\alpha)$. [11]

(ii) Hence or otherwise find the value of α such that $\frac{\mathrm{d}}{\mathrm{d}x}(\tan\alpha) = 0$.

(iii) Find $\frac{\mathrm{d}^2}{\mathrm{d}x^2}(\tan\alpha)$ and hence show that the value of α never exceeds 10°.

d. Find the set of values of x for which $\alpha \geqslant 7^\circ$. [3]

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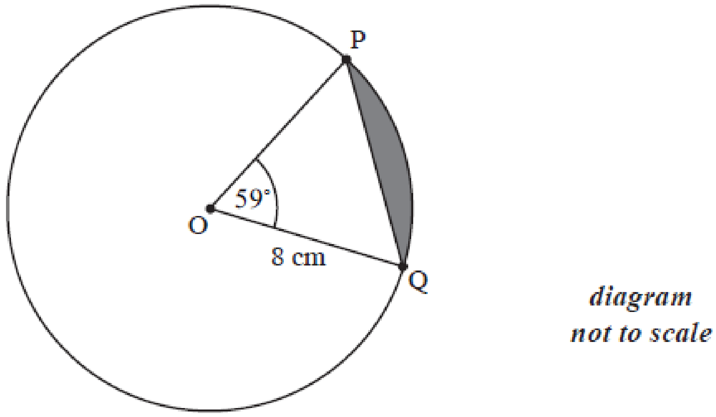
a. Solve the equation $3\cos^2 x - 8\cos x + 4 = 0$, where $0 \leqslant x \leqslant 180^\circ$, expressing your answer(s) to the nearest degree. [3]

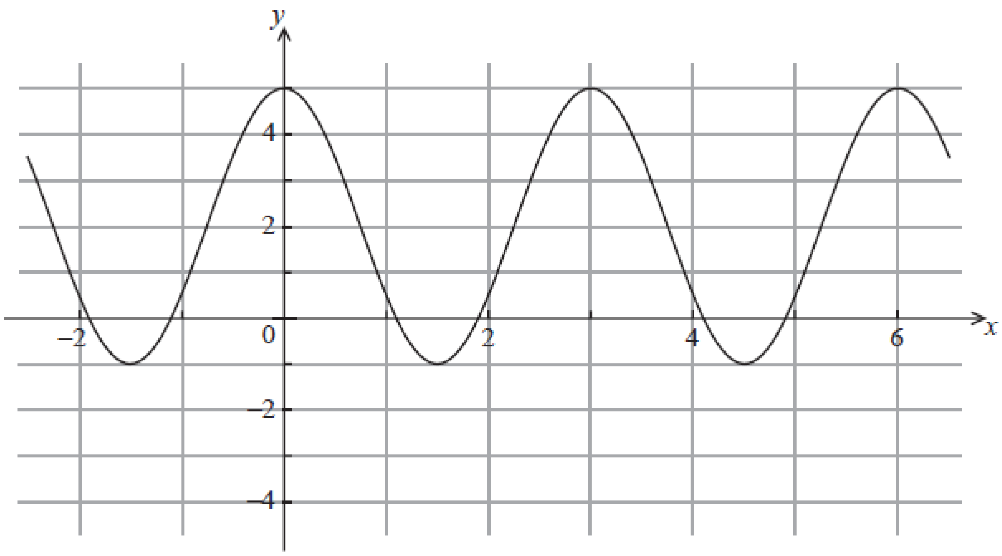
b. Find the exact values of $\sec x$ satisfying the equation $3\sec^4 x - 8\sec^2 x + 4 = 0$. [3]

The points P and Q lie on a circle, with centre O and radius 8 cm, such that $\mathrm{P\hat{O}Q} = 59^\circ$.



Find the area of the shaded segment of the circle contained between the arc PQ and the chord [PQ].

The graph below shows $y = a\cos(bx) + c$.



Find the value of a, the value of b and the value of c.

A system of equations is given by

$$\cos x + \cos y = 1.2$$

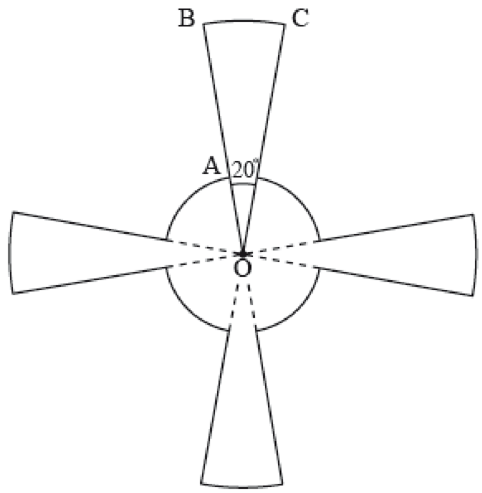
$$\sin x + \sin y = 1.4\ .$$

(a) For each equation express y in terms of x.

(b) **Hence** solve the system for $0 < x < \pi,\ 0 < y < \pi$.

This diagram shows a metallic pendant made out of four equal sectors of a larger circle of radius $\mathrm{OB} = 9\ \mathrm{cm}$ and four equal sectors of a smaller circle of radius $\mathrm{OA} = 3\ \mathrm{cm}$.

The angle $\mathrm{BOC} = 20°$.



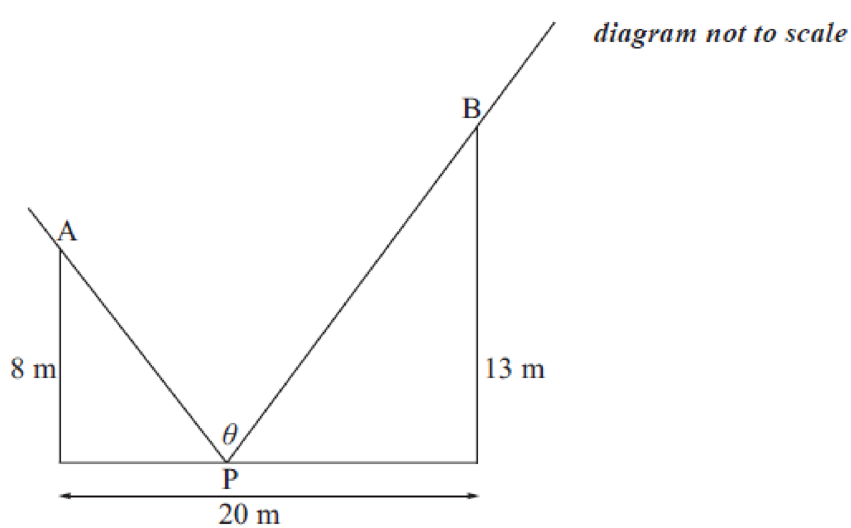
Find the area of the pendant.

A ship, S, is 10 km north of a motorboat, M, at 12.00pm. The ship is travelling northeast with a constant velocity of 20 km hr^{-1}. The motorboat wishes to intercept the ship and it moves with a constant velocity of 30 km hr^{-1} in a direction θ degrees east of north. In order for the interception to take place, determine

a. the value of θ. [4]

b. the time at which the interception occurs, correct to the nearest minute. [5]

A straight street of width 20 metres is bounded on its parallel sides by two vertical walls, one of height 13 metres, the other of height 8 metres. The intensity of light at point P at ground level on the street is proportional to the angle θ where $\theta = \text{A}\hat{\text{P}}\text{B}$, as shown in the diagram.



a. Find an expression for θ in terms of x, where x is the distance of P from the base of the wall of height 8 m. [2]

b. (i) Calculate the value of θ when $x = 0$. [2]

(ii) Calculate the value of θ when $x = 20$.

c. Sketch the graph of θ, for $0 \leqslant x \leqslant 20$. [2]

d. Show that $\frac{d\theta}{dx} = \frac{5(744-64x-x^2)}{(x^2+64)(x^2-40x+569)}$. [6]

e. Using the result in part (d), or otherwise, determine the value of x corresponding to the maximum light intensity at P. Give your answer to four significant figures. [3]

f. The point P moves across the street with speed $0.5\ \mathrm{ms}^{-1}$. Determine the rate of change of θ with respect to time when P is at the midpoint of the street. [4]

The interior of a circle of radius 2 cm is divided into an infinite number of sectors. The areas of these sectors form a geometric sequence with common ratio k. The angle of the first sector is θ radians.

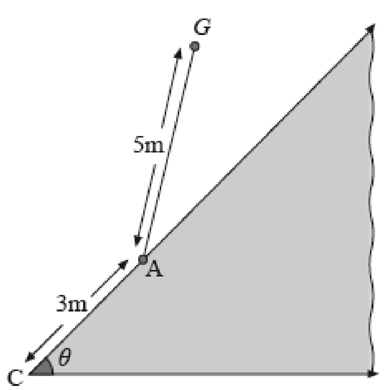
(a) Show that $\theta = 2\pi(1 - k)$.

(b) The perimeter of the third sector is half the perimeter of the first sector.

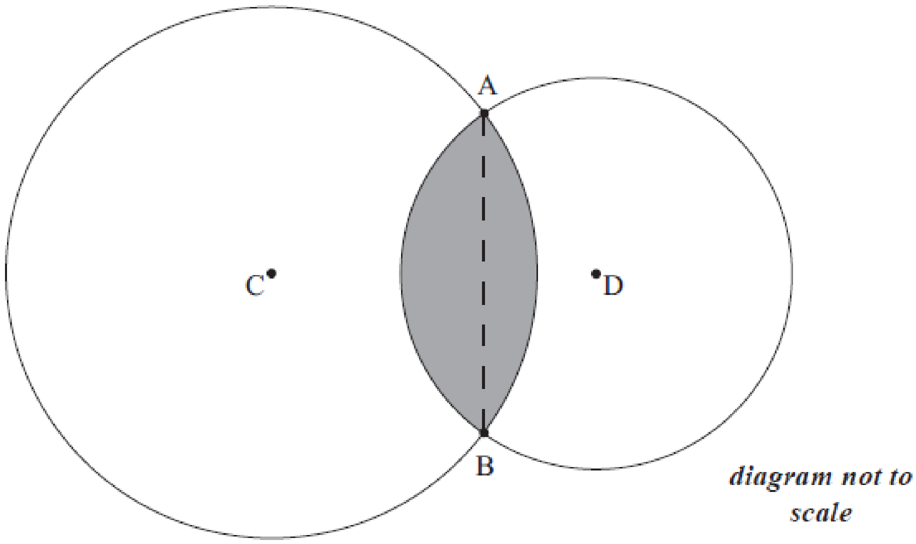
Find the value of k and of θ.

The diagram below shows a fenced triangular enclosure in the middle of a large grassy field. The points A and C are 3 m apart. A goat G is tied by a 5 m length of rope at point A on the outside edge of the enclosure.

Given that the corner of the enclosure at C forms an angle of θ radians and the area of field that can be reached by the goat is $44\ \mathrm{m}^2$, find the value of θ.



The radius of the circle with centre C is 7 cm and the radius of the circle with centre D is 5 cm. If the length of the chord [AB] is 9 cm, find the area of the shaded region enclosed by the two arcs AB.



Compactness is a measure of how compact an enclosed region is.

The compactness, C , of an enclosed region can be defined by $C = \frac{4A}{\pi d^2}$, where A is the area of the region and d is the maximum distance between any two points in the region.

For a circular region, $C = 1$.

Consider a regular polygon of n sides constructed such that its vertices lie on the circumference of a circle of diameter x units.

a. If $n > 2$ and even, show that $C = \frac{n}{2\pi}\sin\frac{2\pi}{n}$. [3]

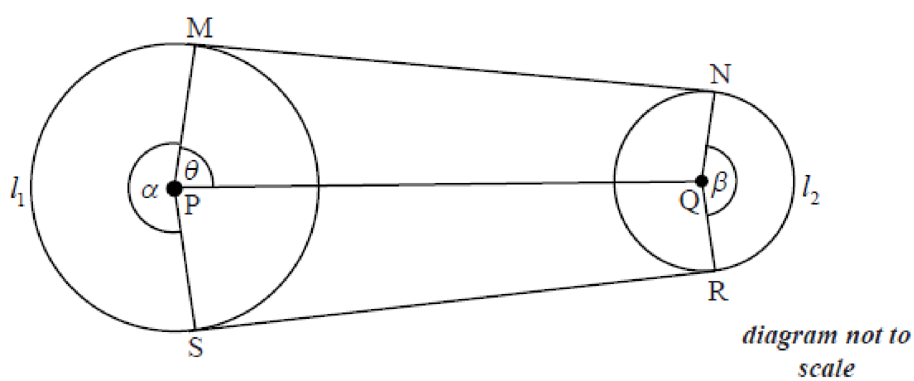
b. If $n > 1$ and odd, it can be shown that $C = \frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)}$. [4]

Find the regular polygon with the least number of sides for which the compactness is more than 0.99.

c. If $n > 1$ and odd, it can be shown that $C = \frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)}$. [1]

Comment briefly on whether C is a good measure of compactness.

Two non-intersecting circles C_1 , containing points M and S , and C_2 , containing points N and R, have centres P and Q where $PQ = 50$. The line segments [MN] and [SR] are common tangents to the circles. The size of the reflex angle MPS is α, the size of the obtuse angle NQR is β , and the size of the angle MPQ is θ . The arc length MS is l_1 and the arc length NR is l_2 . This information is represented in the diagram below.



The radius of C_1 is x , where $x \geqslant 10$ and the radius of C_2 is 10.

(a) Explain why $x < 40$.

(b) Show that cosθ = x −10 50.

(c) (i) Find an expression for MN in terms of x .

(ii) Find the value of x that maximises MN.

(d) Find an expression in terms of x for

(i) α ;

(ii) β .

(e) The length of the perimeter is given by $l_1 + l_2 + \text{MN} + \text{SR}$.

(i) Find an expression, $b(x)$, for the length of the perimeter in terms of x .

(ii) Find the maximum value of the length of the perimeter.

(iii) Find the value of x that gives a perimeter of length 200.

Consider a triangle ABC with $\text{B}\hat{\text{A}}\text{C} = 45.7^\circ$, AB = 9.63 cm and BC = 7.5 cm .

a. By drawing a diagram, show why there are two triangles consistent with this information. [2]

b. Find the possible values of AC . [6]

Points A, B and C are on the circumference of a circle, centre O and radius r . A trapezium OABC is formed such that AB is parallel to OC, and the angle $\text{A}\hat{\text{O}}\text{C}$ is θ , $\frac{\pi}{2} \leqslant \theta \leqslant \pi$.

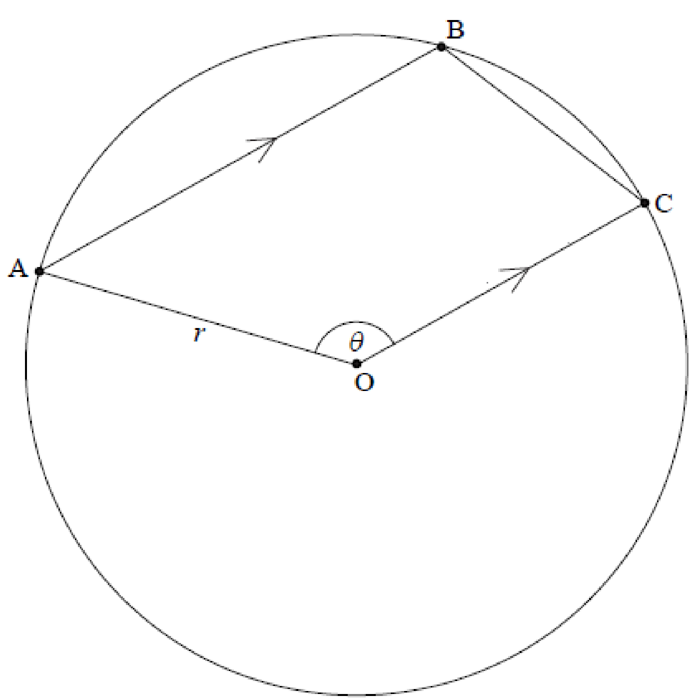


diagram not to scale

(a) Show that angle BÔC is $\pi - \theta$.

(b) Show that the area, T, of the trapezium can be expressed as

$$T = \frac{1}{2}r^2 \sin\theta - \frac{1}{2}r^2 \sin 2\theta.$$

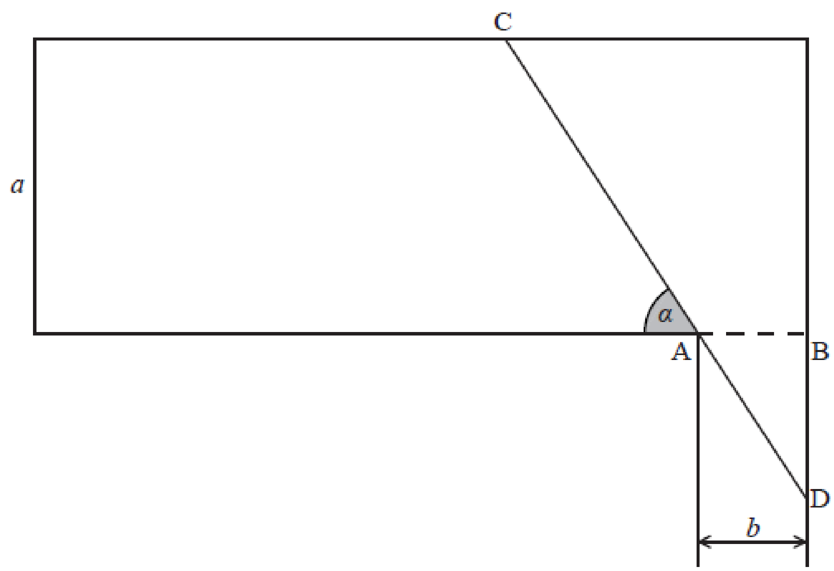
(c) (i) Show that when the area is maximum, the value of θ satisfies

$$\cos\theta = 2\cos 2\theta.$$

(ii) **Hence** determine the maximum area of the trapezium when $r = 1$.

(Note: It is not required to prove that it is a maximum.)

The diagram shows the plan of an art gallery a metres wide. [AB] represents a doorway, leading to an exit corridor b metres wide. In order to remove a painting from the art gallery, CD (denoted by L) is measured for various values of α , as represented in the diagram.



a. If α is the angle between [CD] and the wall, show that $L = \frac{a}{\sin\alpha} + \frac{b}{\cos\alpha}$, $0 < \alpha < \frac{\pi}{2}$. [3]

b. If $a = 5$ and $b = 1$, find the maximum length of a painting that can be removed through this doorway. [4]

c. Let $a = 3k$ and $b = k$. [3]

Find $\frac{dL}{d\alpha}$.

d. Let $a = 3k$ and $b = k$. [6]

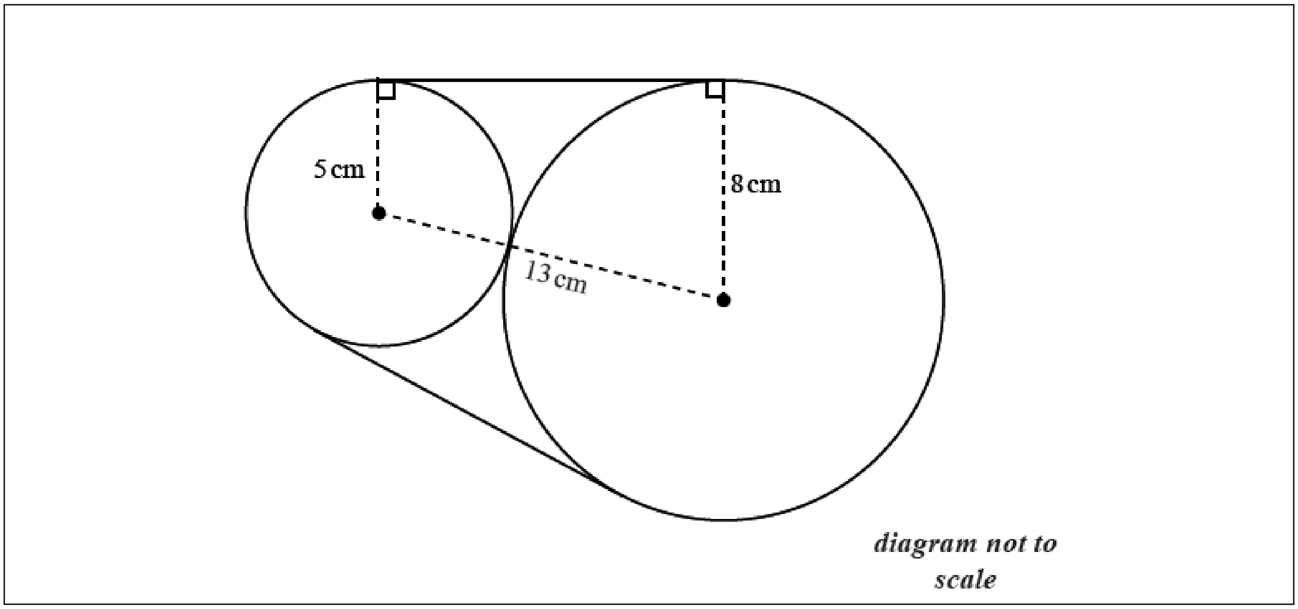
Find, in terms of k , the maximum length of a painting that can be removed from the gallery through this doorway.

e. Let $a = 3k$ and $b = k$. [2]

Find the minimum value of k if a painting 8 metres long is to be removed through this doorway.

Two discs, one of radius 8 cm and one of radius 5 cm, are placed such that they touch each other. A piece of string is wrapped around the discs.

This is shown in the diagram below.



Calculate the length of string needed to go around the discs.